Some characteristics of an equity security next-year impairment

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References

Presentation based of the joint work :

- J. Azzaz, S. Loisel & P.-E. Thérond (2014) Some characteristics of an equity security next-year impairment, *Review of Quantitative Finance and Accounting*, (DOI : 10.1007/s11156-014-0432-x)

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1.1. Framework

- Present standards
- Financial reporting place in the management and development of financial institutions
- Risk (y.c. ORSA for insurance companies)

1.2. Some figures

Table : Financial investments of some insurers in 2011

(Mds €)	Allianz	Axa	CNP Assurances	Generali
Balance Sheet Size	641.472	730.085	321.011	423.057
Total equity	47.253	50.932	13.217	18.120
AFS Assets	333.880	355.126	231.709	175.649
AFS (Funds and equity securities)	26.188	20.636	27.618	20.53

1.3. Overview of IAS 39 impairment disposals

Category	**HTM**	**AFS**		**HFT**
Eligible securities	Bonds	Bonds	Others (stock, funds, etc.)	Everything
Valuation	Amortized cost	Fair Value (through OCI)		Fair Value through P&L
Impairment principle	Event of proven loss	Event of proven loss	Significant or prolonged fall in the fair value	NA
Impairment trigger	Objective evidence resulting from an incurred event (cf. IAS 39 §59)		Two critera (non-cumulative : cf. IFRIC July 2009) : significant **or** prolonged loss in the FV	NA
Impairment Value	Difference between the amortized cost and the revised value of future flows discounted at the original interest rate	In **result** : difference between reported value (before impairment) and the FV		NA
Reversal of the impairment	Possible in specific cases	Possible in specific cases	**Impossible**	NA

1.3. Overview of IAS 39 impairment disposals

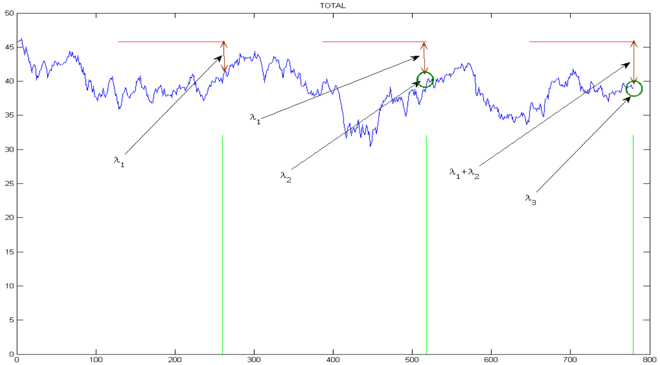

Figure : Illustration : Total Stock price, 2010-2012 ($\alpha = 0.3$, $s = 0.5y$)

1.3. Overview of IAS 39 impairment disposals

Table : Equity instruments impairment parameters used by some financial institutions (2011)

Group	Parameter *significant*	Paramètre *prolonged* (months)	Additional criterion
Allianz	0.2	9	
Axa	0.2	6	
BNP Paribas	0.5	24	0.3 | 12 months
CNP	0.5	36	0.3 | 6 months
Crédit Agricole	0.4	∅	0.2 | 6 months
Generali	0.5	36	
Groupama	0.5	36	
ING	0.25	6	
Scor	0.5	24	0.3 | 12 months
Société Générale	0.5	24	

1.4. Some figures (foll.)

Table : P&L and impairment losses resulting from equity securities classified as AFS 2011

(M€)	Allianz	Axa	CNP Assurances	Generali
Result	2804	4516	1141	1153
Impairment losses on AFS funds and equity securities	-2487	-860	-1600	-781

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2.1. Notations

Main notations :

- $S = (S_t)_{t \geq 0}$ the stock price process
- t_a the acquisition date
- $\lambda = (\lambda_s)_{s \in \{[t_a]+1, [t_a]+2, \ldots\}}$ the successive impairment losses (may be nil)
- $\Lambda(S, t_a, t) = \sum_{s=\lfloor t_a \rfloor +1}^{t} \lambda(S, t_a, s)$ the sum of pas impairment losses
- $\Omega(S, t_a, t)$ the amount in OCI resulting from S at time t

At each reporting date t, the balance sheet equilibrium property leads to :

$$S_t - S_{t_a} = \Omega(S, t_a, t) + \Lambda_t.$$

2.2. Impairment triggers

A necessary condition for considering an impairment loss at time $t+1$ is :

$$\begin{cases} S_{t+1} \leq (1-\alpha) S_{t_a}, \text{or}; \\ \forall u \in \left] t+1-s, t+1 \right], S_u \leq S_{t_a}, \end{cases}$$

where α and s are determined by the reporting entity. α represents the relative level of fall in fair value since the acquisition date corresponding to *significant* decline, s represents the minimum period before the financial reporting date that leads to consider that the decline is *prolonged*.

Moreover, there is an effective impairment loss if , in addition :

$$S_{t+1} \leq S_{t_a} - \Lambda_t.$$

Then, the impairment loss λ_{t+1} is given by :

$$\lambda_{t+1} = S_{t_a} - \Lambda_t - S_{t+1} = K_t - S_{t+1}.$$

2.3. Probability and amount of impairment loss

Let us denote J_{t+1} the probability of an effective impairment loss at reporting date $t+1$:

$$J_{t+1} = (S_{t+1} \leq (1-\alpha)S_{t_a}, S_{t+1} \leq K_t) \bigcup \left(\max_{t+1-s \leq u \leq t+1} S_u \leq S_{t_a}, S_{t+1} \leq K_t \right),$$

By introducing $m_t = \min((1-\alpha)S_{t_a}, K_t)$, we have :

$$\begin{aligned} \mathbb{P}_t\left[J_{t+1}\right] = \mathbb{P}_t\left[S_{t+1} \leq m_t\right] + \mathbb{P}_t\left[\max_{t+1-s \leq u \leq t+1} S_u \leq S_{t_a}, S_{t+1} \leq K_t\right] \\ - \mathbb{P}_t\left[\max_{t+1-s \leq u \leq t+1} S_u \leq S_{t_a}, S_{t+1} \leq m_t\right]. \end{aligned}$$

2.3. Probability and amount of impairment loss

Similarly, the impairment loss at time $t+1$ is given by :

$$\lambda_{t+1} = (K_t - S_{t+1})^+ \mathbf{1}\left\{\max_{t+1-s\leq u\leq t+1} S_u \leq S_{t_a} \cup S_{t+1} \leq (1-\alpha)S_{t_a}\right\},$$

with $K_t = S_{t_a} - \Lambda_t$.
This expression can be expressed as a sum of three terms :

$$\lambda_{t+1} = X_{t+1} + Y_{t+1} - Z_{t+1},$$

avec

- $X_{t+1} = (K_t - S_{t+1})^+ \mathbf{1}\left\{\max_{t+1-s\leq u\leq t+1} S_u \leq S_{t_a}\right\}$,
- $Y_{t+1} = (K_t - S_{t+1})^+ \mathbf{1}\left\{S_{t+1} \leq (1-\alpha)S_{t_a}\right\}$, and
- $Z_{t+1} = (K_t - S_{t+1})^+ \mathbf{1}\left\{\max_{t+1-s\leq u\leq t+1} S_u \leq S_{t_a}\right\} \mathbf{1}\left\{S_{t+1} \leq (1-\alpha)S_{t_a}\right\}$.

2.3. Probability and amount of impairment loss

The three terms could be seen as payoffs of options of S :

- the first one : a *rear-end up-and-out* put option (cf. Hui (1997))
- the second one : a traditional European put option,
- the third one corresponding to the sum of a *rear-end up-and-out* put option and a compensation amount.

The main objective of our work is to exhibit some characteristics of future impairment losses (with a one-year horizon) for risk management purposes (prediction, risk measures and decisions), the following results are obtained :

- using option theory ;
- under the real-world probability measure.

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3.1. Model and results overview

Considering a Black & Scholes framework (i.e. a geometric brownian motion), we obtain closed formulas for (one-year horizon) :

- the probability that some impairment occurs,
- the expectation of impairment losses,
- the cumulative distribution function (c.d.f.) of impairment losses.

3.2. Main results

Theorem (Impairment probability)

The probability to recognize an impairment at future time $t+1$, given the information $\mathcal{F}_t$ at time t, is given by

$$\mathbb{P}_t\left[J_{t+1}\right] = \left(\frac{S_{t_a}}{S_t}\right)^{k_1-1}\left[\Psi_\rho\left(C, D(K_t)\right) - \Psi_\rho\left(C, D(m_t)\right)\right] + \Phi\left(-A(K_t)\right) + \Psi_\rho\left(B, A(K_t)\right) - \Psi_\rho\left(B, A(m_t)\right),$$

where Φ denotes the c.d.f. of a standard normal distribution, and Ψ_ρ is the bivariate normal distribution function : for all x, y,
$\Psi_\rho(x,y) = \mathbb{P}_t\left[X \leq x, Y \leq y\right]$ where (X, Y) is a Gaussian vector with standard marginals and correlation ρ.

3.2. Main results

Other terms, for $x \in \{m_t, K_t\}$:

- $A(x) = \frac{\ln(S_t/x)+\mu}{\sigma} - \frac{\sigma}{2}$, $A'(x) = A(x) + \sigma$,
- $B = \frac{\ln(S_t/S_{t_a})+\mu(1-s)}{\sigma\sqrt{(1-s)}} - \frac{\sigma\sqrt{(1-s)}}{2}$, $B' = B + \sigma\sqrt{(1-s)}$,
- $C = \frac{\ln(S_{t_a}/S_t)+\mu(1-s)}{\sigma\sqrt{(1-s)}} - \frac{\sigma\sqrt{(1-s)}}{2}$, $C' = C + \sigma\sqrt{(1-s)}$,
- $D(x) = \frac{\ln(S_{t_a}^2/S_t x)+\mu}{\sigma} - \frac{\sigma}{2}$, $D'(x) = D(x) + \sigma$,
- $k_1 = \frac{2\mu}{\sigma^2}$,

3.2. Main results

Theorem (Impairment loss expectation)

The expectation of next-year impairment, given the information $\mathcal{F}_t$ at time t, is given by

$$\begin{aligned}\mathbb{E}_t\left[\lambda_{t+1}\right] = & \, S_t e^{\mu}\left(\frac{S_{t_a}}{S_t}\right)^{k_1-1}\left[\Psi_{-\rho}\left(C',-D'(K_t)\right)-\Psi_{-\rho}\left(C',-D'(m_t)\right)\right] \\ & + S_t e^{\mu}\left[\Psi_{\rho}\left(-B',-A'(m_t)\right)-\Phi\left(-A'(m_t)\right)-\Psi_{\rho}\left(-B',-A'(K_t)\right)\right] \\ & + K_t\left[\Psi_{\rho}\left(-B,-A(K_t)\right)+\Phi\left(-A(m_t)\right)-\Psi_{\rho}\left(-B,-A(m_t)\right)\right] \\ & + (K_t-m_t)\left(\frac{S_{t_a}}{S_t}\right)^{k_1-1}\left[\Phi\left(-D(m_t)\right)-\Psi_{\rho}\left(-C,-D(m_t)\right)\right] \\ & - K_t\left(\frac{S_{t_a}}{S_t}\right)^{k_1-1}\Psi_{-\rho}\left(C,-D(K_t)\right)+m_t\left(\frac{S_{t_a}}{S_t}\right)^{k_1-1}\Psi_{-\rho}\left(C,-D(m_t)\right),\end{aligned}$$

where all constant numbers, variables and parameters are defined in Theorem 1.

3.2. Main results

Two sensitivity results :

Theorem (Probability sensitivity)

The probability is decreasing according to α, μ, Λ and s.
Moreover, the probability is convex according to α, μ, μ and s.

Theorem (Expectation sensitivity)

The expected impairment loss is decreasing according to α, μ, Λ and s, non-decreasing with σ. Moreover, it is convex according to α, μ, σ and Λ, and concave with s.

3.2. Main results

Theorem (Cumulative distribution function of impairment loss)

The cumulative distribution function of the next-year impairment loss, given the information $\mathcal{F}_t$ available at time t, is given by :

$$\mathbb{P}_t\left[\lambda_{t+1} \leq l\right] = \begin{cases} \left(1 - \mathbb{P}_t\left[J_{t+1}\right]\right) + \Phi\left(A(K_t - l)\right) - \Phi\left(A(K_t)\right) \\ + \left(\frac{S_{t_a}}{S_t}\right)^{k_1 - 1}\left[\Psi_\rho\left(C, D(K_t)\right) - \Psi_\rho\left(C, D(K_t - l)\right)\right] \\ +\Psi_\rho\left(B, A(K_t)\right) - \Psi_\rho\left(B, A(K_t - l)\right) & , 0 \leq l \leq K_t - m_t, \\ \Phi\left(A(K_t - l)\right) & , K_t - m_t < l \leq K_t, \end{cases} \quad (1)$$

with the same notations and variables as in 1.

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4.1. Sensitivities

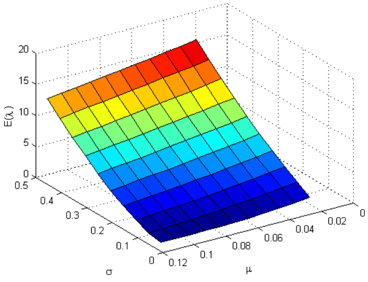

Figure : Average next-year impairment as a function of μ and σ (left), and of α and Λ (right).

4.2. Illustrations

Table : One-year expected loss for TOTAL according to past impairment losses

Total. $S_{t_a} = 45.795$

S_t	Λ	$\mathbb{P}_t[J_{t+1}]$	$\mathbb{E}_t[\lambda_{t+1}]$	$VaR(80\%)$	$VaR(95\%)$	$VaR(99.5\%)$
38.42	2.28975	0.5509	4.0699	10.7697	16.2236	21.402
	4.5795	0.5075	3.3007	8.4799	13.9338	19.1123
	22.8975	0.0078	0.0124	0	0	0.7943
	34.34625	0	0	0	0	0

4.2. Illustrations

Table : Impact of impairment parameters (Axa et Generali) for five stocks

	Axa		Generali	
	$\mathbb{P}_t\left[J_{t+1}\right]$	$\mathbb{E}_t\left[\lambda_{t+1} \mid J_{t+1}\right]$	$\mathbb{P}_t\left[J_{t+1}\right]$	$\mathbb{E}_t\left[\lambda_{t+1} \mid J_{t+1}\right]$
BNP Paribas	0.3331	21.3545	0.0698	33.8002
Bouygues	0.2762	10.3336	0.011	20.3283
Carrefour	0.2851	8.7095	0.0162	16.4673
Pernod Ricard	0.2374	13.1027	0.00076	32.1938
Total	0.2365	9.7935	0.00069	24.2181

What's next ?

Work in progress (DéCAF project) :

- multi-period framework ;
- other stock price models ;
- portfolio assessment ;
- expected credit losses (IFRS 9).

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