Technical reserves and solvency capital of insurance company: how to use the Value-at-Risk?

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Introduction

The advent of the future European prudential framework (Solvency II) and, to a lesser extent, of the phase II of the IFRS dedicated to the insurance contracts, will systematize the use of the Value-at-Risk (VaR) risk measure in insurance.

We can distinguish two quantitative requirements:

- the technical provisions will have to be sufficient to pay the claims with a 75 % probability (a CoC approach is too envisaged);
- the solvency capital will have to be calibrated to control the ruin of the insurer with a probability higher than 99.5%.

Introduction

The two new quantitative requirements framed by Solvency II, both refer to a VaR, but their nature strongly differs.

The requirement on the technical reserves will not raise any major problem. Indeed,

- the data observed and used by the insurers are numerous and
- the stochastic methods to calculate the provisions are robust.

Moreover, the percentile that has to be estimated is not very high, and thus the study focuses on the core of the distribution where the information is reliable.

Introduction

The computation of the Solvency Capital Requirement (SCR) is not so simple:

- the insurer does not observer directly the interest variable (profit),
- the percentile to estimate is very high (99.5%).

Operationally the SCR will be determined using:

- a standard formula (cf. QIS 3), or
- an internal model.

1. Internal model methodology

The SCR computation problematic relies on the fact that:

- only few data are available (at best, a few years of observation of the profit, for instance),
- not any in the appropriate area of the distribution.

To compute the SCR, the insurer has to build an internal model which enables to simulate the financial position of the company within 1 year.

Then, using simulations techniques, the insurer will dispose of (simulated) realizations of the interest variable in order to estimate the 99.5% VaR.

1. Internal model methodology

Each stage of the internal model building entails risks:

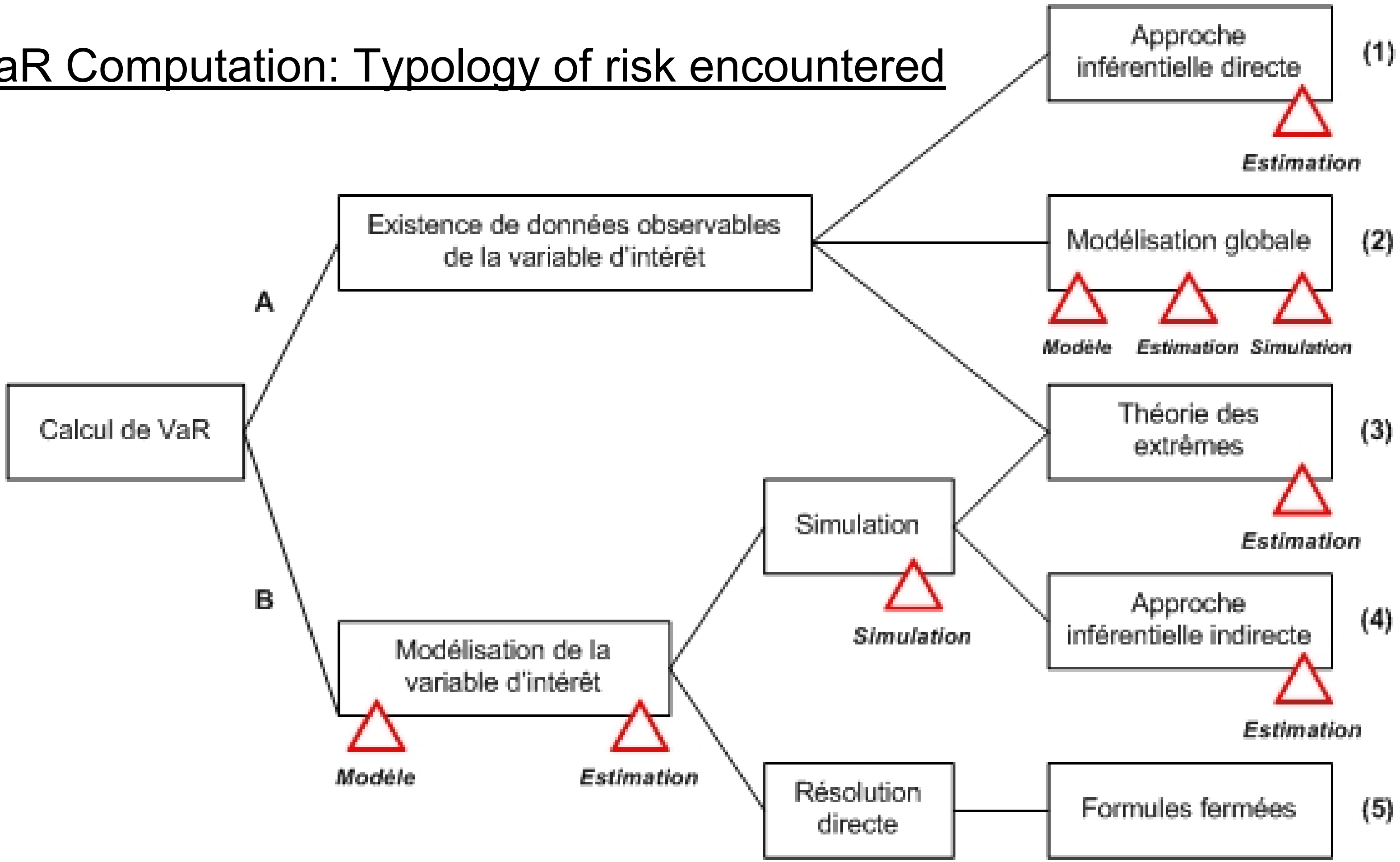
- a modelling risk: the model that is used provides an image of the reality that is not perfect; moreover, the models usually used to represent both the assets and the liabilities tend to underestimate the extreme situations;
- an estimation risk : the estimated parameters that feed the model are spoilt by an error. Its consequence might be severe when the model lacks robustness;
- a simulation risk: the profit's distribution will usually be estimated using a simulation and thus will only be approximate.

1. Internal model methodology

Furthermore, when it comes to the estimation of a percentile of high level (99.5% VaR), and knowing that the shape of the profit's distribution is generally hard to fit to a global parametric model, we ought to turn the extreme value techniques to finally calculate the risk measurement; making appear a new estimation risk on the tail distribution that is simulated.

1. Internal model methodology

VaR Computation: Typology of risk encountered



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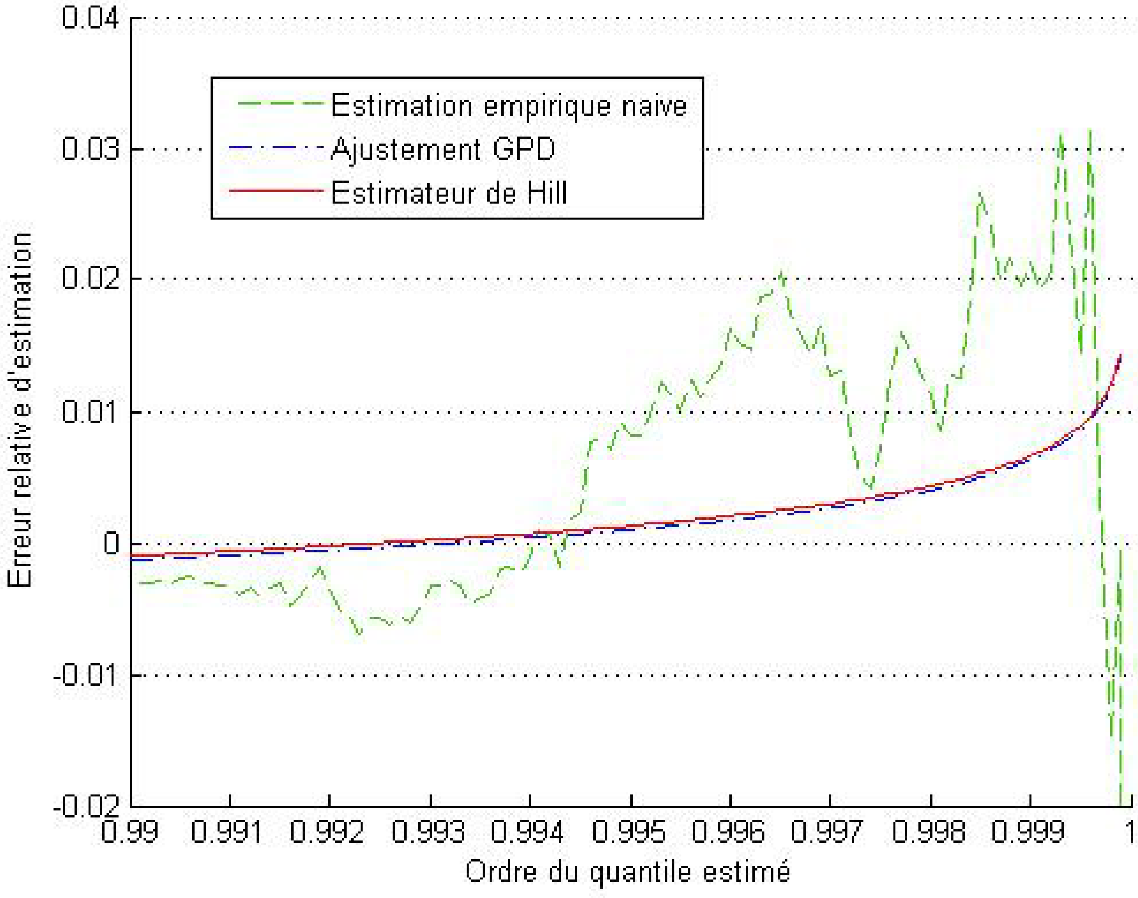
2. Extreme VaR estimation

One can adopt several approaches:

- natural empirical estimation,
- bootstrap methods (classical, percentile, BCa, etc.)
- EVT (Hill's estimator, POT).

The EVT gives the most efficient estimations.

2. Extreme VaR estimation



3. Robustness of the SCR

Let us consider the simplified internal model inspired by the works of Deelstra and Janssen (1998) in which:

- the losses of the year follow a log-normal distribution and are paid at the end of the year; $B \sim \mathcal{LN}(m, s)$
- the financial return is assumed to be gaussian. $\rho \sim \mathcal{N}(\mu, \sigma)$
- these r.v. are supposed to be independent.

Let's denote a_0 the initial amount of assets the insurer must have in order to be solvable at the end of the year with a probability of 1-α.

3. Robustness of the SCR

$$a_0 = \min\left\{a > 0 \mid \Pr\left[ae^{\rho} \geq B\right] > 1 - \alpha\right\}$$

$$a_0 = \exp\left\{m - \mu + \Phi^{-1}(1-\alpha)\sqrt{s^2 + \sigma^2}\right\}$$

We can study the sensibility of the SCR to the parameters of the basic models:

$$\frac{1}{a_0}\frac{\partial a_0}{\partial m} = 1 \qquad \frac{1}{a_0}\frac{\partial a_0}{\partial s} = \frac{\Phi^{-1}(1-\alpha)}{\sqrt{1+\sigma^2/s^2}}$$

An error of 1% on *s* leads to an error of $\Phi^{-1}(1-\alpha)\Big/\sqrt{1+\sigma^2/s^2}$ on a_0.

With a 99.5% VaR, when $s \simeq \sigma$ the error is 1.82 times higher!

3. Robustness of the SCR

The using of simulation techniques can generate different kinds of error:

- fluctuations of sampling linked to the finite numbers of draw that are done;
- discretization bias when transforming a continuous model in its discrete versions;
- errors associated to the approximations used by some of the inversing techniques;
- bias incurred by an inappropriate choice of the generator of random numbers.

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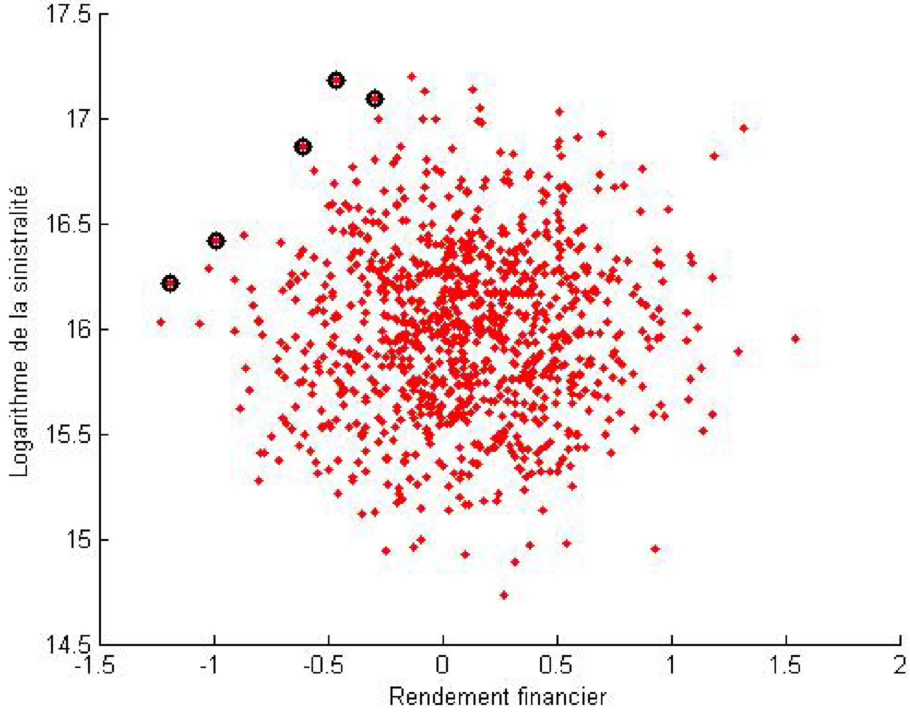
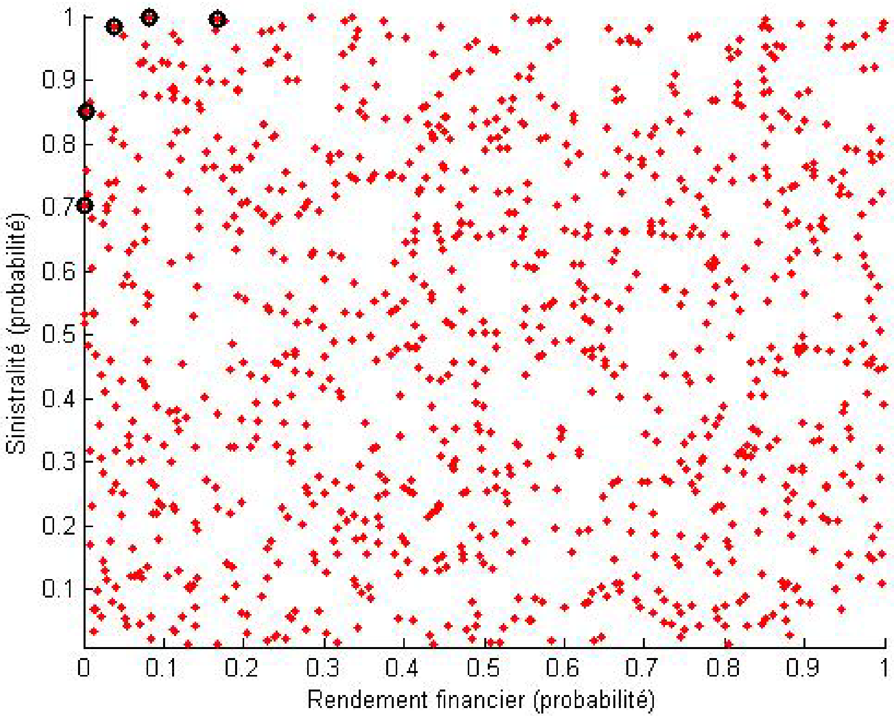
3. Robustness of the SCR

The purpose of an internal model implies to be able to represent in a good manner the tail values of the basic variables (sinistrality, financial returns, etc.)



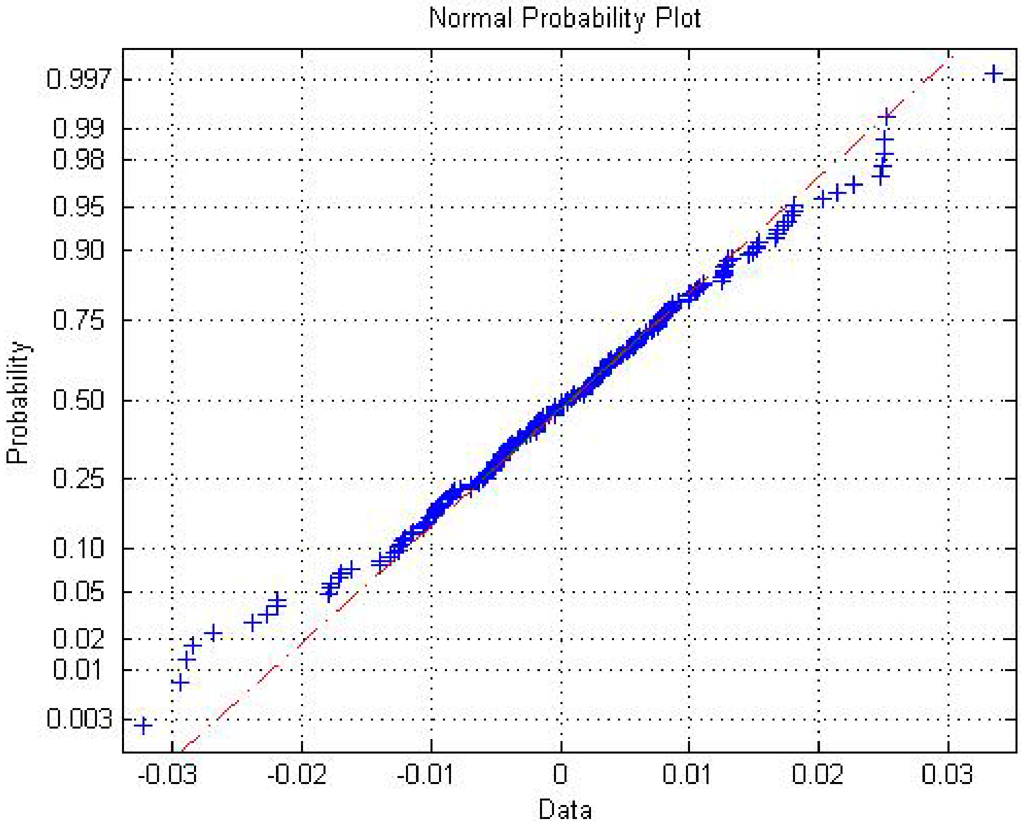
3. Robustness of the SCR



The worst scenarios for the company come from the extreme values of the basic variables ⇒ the adequacy to the basic model has to be checked in the tail distribution.

3. Robustness of the SCR

Example: modelling the TOTAL stock daily return



The global adequacy is good (Jarque-Béra, Lilliefors).

Error of 7.5% for the 0.5% percentile.

3. Robustness of the SCR

The normal model underestimates the probability to get bad returns.

One could imagine to use non-parametric models but it's not satisfactorily in an internal model context.

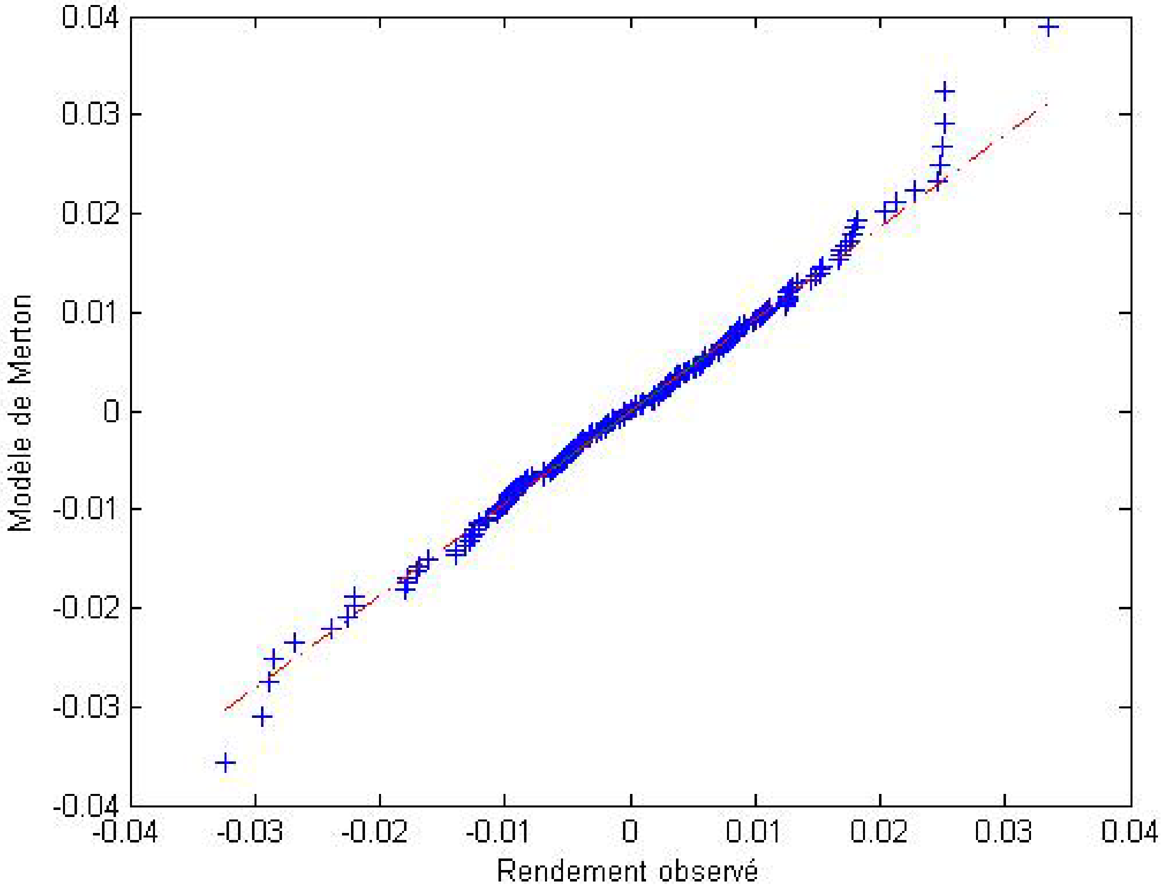
We have to find models that fit better with the tail distribution.

For example, for the financial return of the TOTAL stock, this mono-periodic version of the Merton (1976) process

$$\rho = \mu + \sigma_0 \varepsilon_0 + \sigma_u \sum_{i=1}^{N} \varepsilon_i$$

enables to achieve this objective.

3. Robustness of the SCR



Conclusion

Solvency II will systemize the use of VaR in insurance.

The SCR is defined in reference to the tail distribution of a variable which cannot be observed.

To compute this SCR, the insurers will have to fulfil the standard formula and, as an alternative, to build an internal model.

The level of the percentile retained asks the question of the robustness of the criterion. The process to obtain the solvency capital is long and many errors (model, estimation, simulation, etc.) may occur.

The validation process of this kind of models will be crucial to ensure the reliability of the SCR.

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