MODEL RISK AND DETERMINATION OF ECONOMIC CAPITAL IN THE SOLVENCY 2 PROJECT

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Introduction

The Solvency 2 project (see Commission européenne [2003], [2004] and AAI [2004]) that is still in development modifies deeply the fixing rules of the level of equity in insurance. This project introduces as explicit criterion the control of total risk supported by the company. This risk will have to be quantified through the ruin probability at a time horizon of one year.

The retained level of 99.5% implies the requirement to assess suitably a high-order quantile of the interest distribution (generally and in our case, the excess distribution or the distribution of the asset-liability margin). This problematical point is widely built up in the financial literature that is confronted with these questions since the Basel II accords in the banking area. For instance, we can quote Robert [1998] or Gauthier and Pistre [2000].

Introduction

In this new insurance context, the classic asset/liability modellings that accredit a limited attention at the tail distribution modelling can be proved a penalizing point, because they lead at a low-level representation of extreme values. For instance, this point is illustrated for the modellings of asset in Ballotta [2004] in case of hidden options in life insurance contracts, and in Planchet and Therond [2005] in the framework of mono-periodic simplified model in non-life insurance for the determination of the target capital and asset allocation. Thérond and Planchet [2007] draw the intention to the extent of extremes in the determination of Solvency Capital Requirement (SCR).

In this present article, we develop this point of view in disturbing a model of simple reference in making heavy its tail distribution. It is shown that is possible to obtain situations in which the basic model underestimates significantly the Solvency Capital Requirement, while being not easily discernible statistically with the disturbed model if a detailed attention is not paid to the extreme values.

Description of the model

we suppose the positive random X is defined by the following survival function:

$$S_X(x) = \begin{cases} S_0(x) & x \le m \\ \left(\dfrac{x}{m}\right)^{-\alpha} S_0(m) & x > m \end{cases}$$

The distribution is « almost » log-normal, with Pareto tail.

$$P(X > x | X > m) = \frac{P(X > x)}{P(X > m)} = \frac{S_X(x)}{S_X(m)} = \left(\frac{x}{m}\right)^{-\alpha}$$

Quantile function :

$$F^{-1}(p) = x_p = m \times \left(\frac{1-p}{S_0(m)}\right)^{-1/\alpha}$$

Un modèle mélangé

We thus have to compare :

$$x_p^{MEL} = \mathbf{exp}\left(\mu + \sigma\phi^{-1}\left(p_0\right)\right) \times \left(\frac{1-p}{1-p_0}\right)^{-1/\alpha} \qquad x_p^{LN} = \mathbf{exp}\left(\mu + \sigma\phi^{-1}\left(p\right)\right)$$

$$r\left(\alpha\right) = \frac{x_p^{MEL}}{x_p^{LN}} = \mathbf{exp}\left(\sigma\left(\phi^{-1}\left(p_0\right) - \phi^{-1}\left(p\right)\right)\right) \times \left(\frac{1-p}{1-p_0}\right)^{-1/\alpha}$$

We will be confronted with the situation of model risk in the case where despite a value $r\left(\alpha\right) >> 1$, a sample derived from the blended model would be difficult to differentiate with a lognormal sample. The lognormal model is very widespread in insurance and in particular, it is on this model that were gauged a part of parameters of the standard formula described in QIS 3. We are going to pay particular attention to examine this situation in the continuation of this paper.

Estimation of the model paramaters

The estimation of parameters can be performed by the maximum likelihood method

$$l\left(x_1,..,x_n;\mu,\sigma,m,\alpha\right)=cste-(k-1)\ln(\sigma)-\frac{1}{2}\sum_{i=1}^{k-1}\left(\frac{\ln\left(x_{(i)}\right)-\mu}{\sigma}\right)^2$$

$$+(n-k+1)\ln(\alpha)+\alpha(n-k+1)\ln(m)-\alpha\sum_{i=k}^{n}\ln\left(x_{(i)}\right)+(n-k+1)S_0(m)$$

with

$$k=\min\left\{i;x_{(i)}\geq m\right\}$$

$$\max_{(\mu,\sigma,m,\alpha)} l\left(x_1,..,x_n;\mu,\sigma,m,\alpha\right)=\max_{m}\max_{(\mu,\sigma,\alpha)} l\left(x_1,..,x_n;\mu,\sigma,m,\alpha\right)$$

$$\hat{\mu}=\frac{1}{k-1}\sum_{i=1}^{k-1}\ln x_{(i)} \qquad \hat{\sigma}=\sqrt{\frac{1}{k-1}\sum_{i=1}^{k-1}\left(\ln x_{(i)}-\hat{\mu}\right)} \qquad \hat{\alpha}=\frac{n-k+1}{\sum_{i=k}^{n}\ln\left(\frac{x_{(i)}}{m}\right)}$$

Estimation of the model paramaters

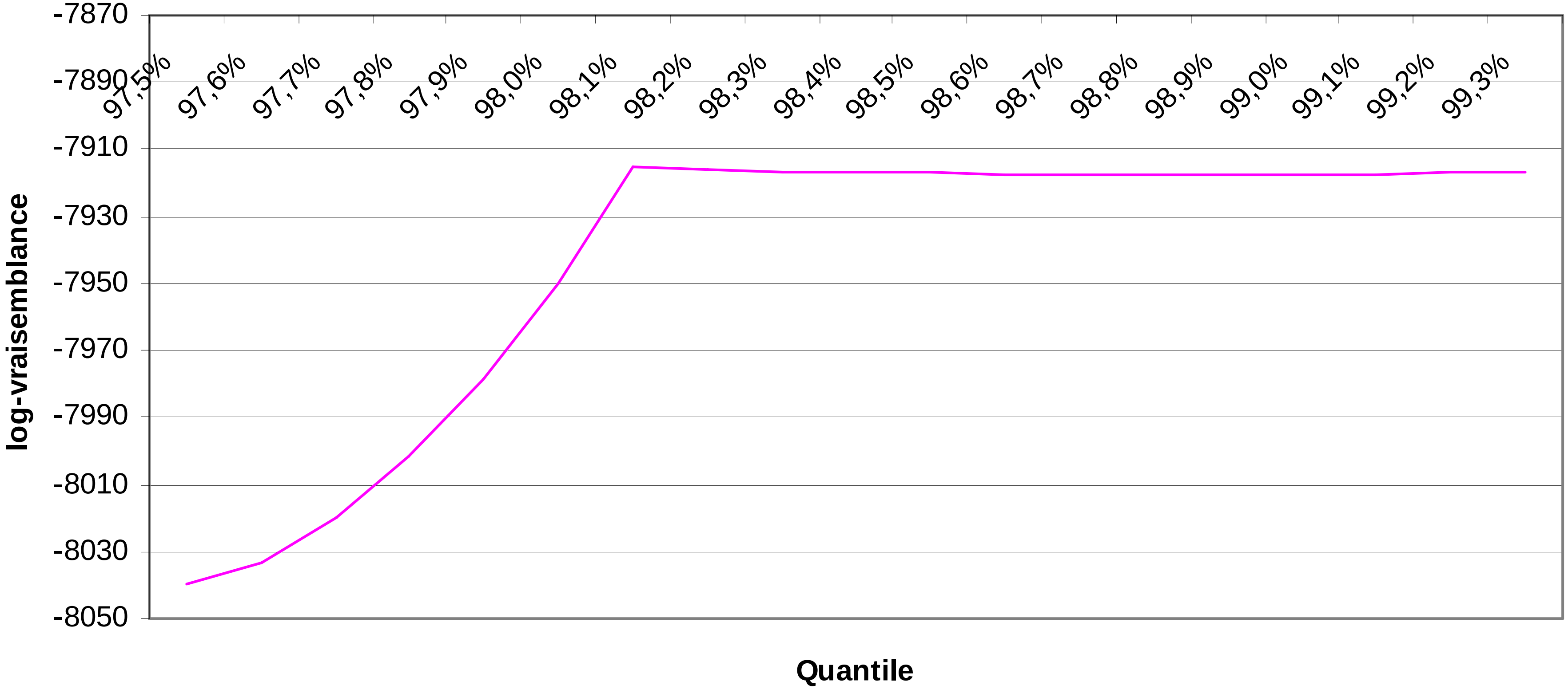
It remains to eliminate *m*, unknown, in the above equation. In practice we can proceed in the following way:

- we fix k (while starting for example by $k = 95\% \times n$)
- we calculate $\hat{\mu}$ and $\hat{\sigma}$
- we calculate $\hat{m} = \mathbf{exp}\left(\hat{\mu} + \hat{\sigma}\phi^{-1}\left(\frac{k}{n}\right)\right)$
- the estimator (pseudo maximum likelihood) of tail parameter is given by the expression:

$$\hat{\alpha} = \frac{n-k+1}{\sum_{i=k}^{n} \mathbf{ln}\left(\frac{x_{(i)}}{\hat{m}}\right)}$$

Estimation of the model paramaters

Identification of m :



Numerical application

The simulation of a sample resulting from the blended distribution can be obtained simply in the following way:

- drawing of a value u uniformly distributed on $[0,1]$
- if $u > p_0$, drawing of x in the Pareto distribution with parameters (m,α)
- else, drawing of x in distribution .

$$S(x) = \frac{S_0(x) - S_0(m)}{1 - S_0(m)}$$

This last drawing can be carried out with a rejection method: we make a drawing in the lognormal distribution, and we refuse it if the obtained value is higher than m.

Application

Identification of the extreme values

We notice that if we fix a probability , then the probability that the p-order quantile of the lognormal distribution is exceeded in the blended distribution is:

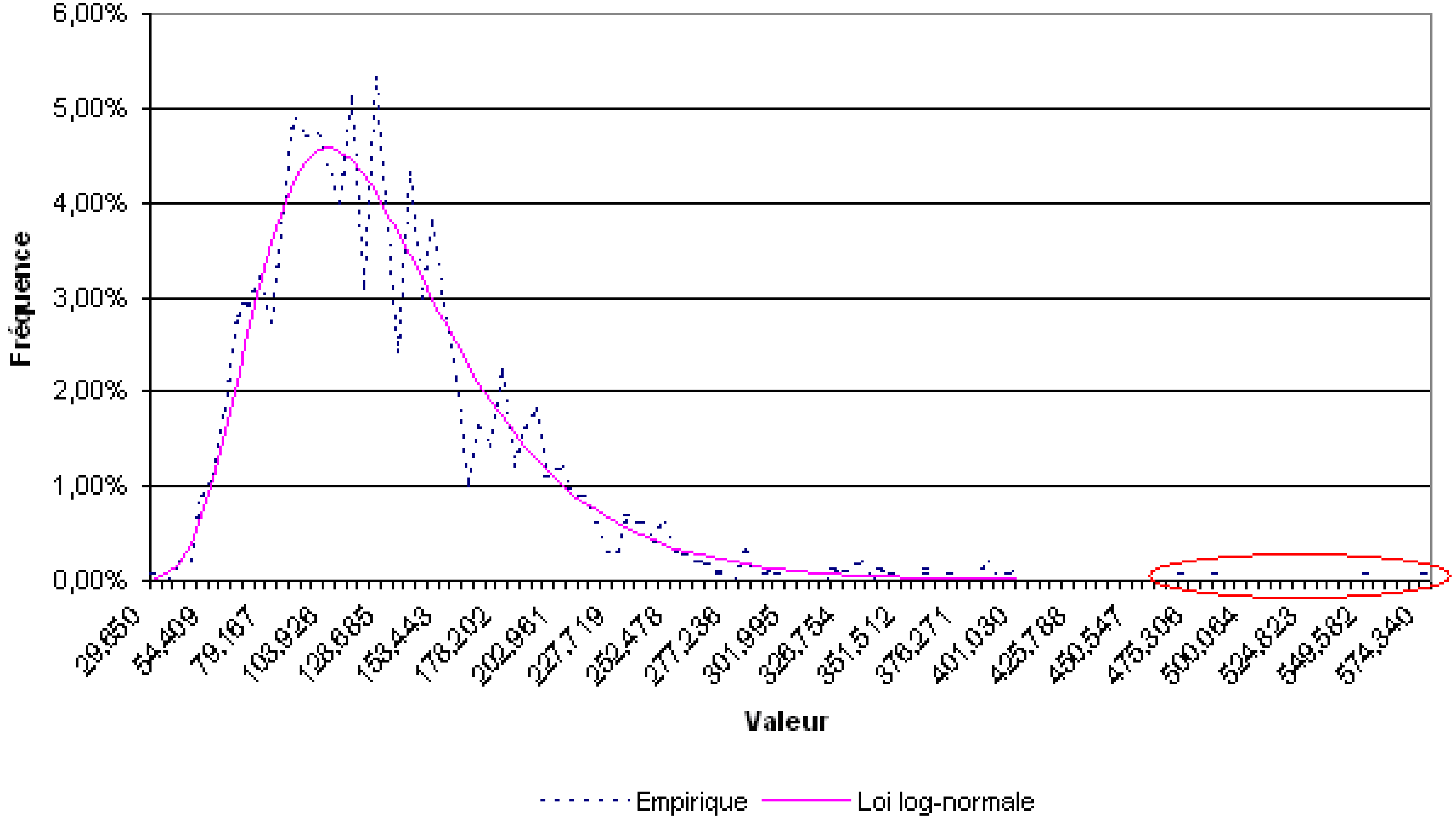
$$\pi(p) = 1 - \left(\frac{\exp\left(\mu + \sigma\phi^{-1}(p)\right)}{m} \right)^{-\alpha} S_0(m)$$

With $p = 99.8\,\%$ then $\pi(p) = 0.50\,\%$

As a consequence, on a sample of 1000 values, we will get on average two values which exceed , whereas there will be 5 values which will exceed this threshold if the subjacent distribution is the blended distribution. As the number of values exceeding a threshold u is approximately normal we obtain:

$$P(N_u \geq k) \approx 1 - \phi\left(\frac{k - nS(u)}{\sqrt{nS(u)\left(1 - S(u)\right)}} \right)$$

Identification of the extreme values



Numerical application

The values obtained on a "typical" sample arise in the following way:

	Estimation	Theoretical	
μ	4.958	5	
σ	0.386	0.4	
γ	317.097799	353.553971	
α	3.475	3.9	
Estimated ratio =	117%	113%	
Solvency capital requirement			
SCR LN	416.00	415.85	0.0%
SCR mélangé	451.29	468.59	-3.7%

The estimation of SCR in lognormal sample is relatively robust in the case of a sample of size 1000. However, we observe an underestimation of the capital in the case of the blended model. In the end, we can retain if the data result from the blended model, the fact of considering that they are really issued from a lognormal sample leads to an important underestimation of the capital requirement. Moreover within the framework of the well-specified model, the estimation still leads to a light underestimation.

Conclusion

The results presented here, within a very simplified framework, underline once again the lack of robustness that is inherent in the criterion of fixing of the Solvency Capital Requirement in the project Solvency 2 project.

So it seems essential to us that the implementation methods of the ruin probability criterion are clarified in the long term and notably that the constraints on the modelling of the tail distribution are specified within the framework of an internal model. These constraints must be expressed on three levels: for the asset modelling, for the liability modelling, and finally within the framework of the exploitation of the empirical distribution of a asset-liability margin simulated from "way out" of the model.

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