Impact of the asset jumps in insurance : IFRS / Solvency II

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Context (1/2)

The aim of this paper is to propose an operational statistical model with jumps for a risky asset, by generalizing the Black & Scholes model.

The B&S model is a current reference for experts in insurance, but in the new context generated by :

- the accounting standards changes (IFRS) and
- prudential changes (Solvency II),

this model results in an underestimate of the risk of placement and therefore of the provisions or the solvency required capital of the company.

Context (2/2)

The statistical procedures for estimating the parameters of the selected model (Merton’s model) are presented and illustrated on real data.

Then the model is applied in problems of:

- pricing of options (> IFRS 4),
- determination of a SCR (> Solvency II) in a very simple framework.

We show that the results may be very different with or without jumps, under a fixed level of global volatility.

The model

The asset dynamic is expressed as follows :

$$S(t) = S(0)\exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma B_t + \sum_{k=1}^{N_t} U_k\right\}$$

Where $B = (B_t)_{(t\geq 0)}$ is a brownian motion, $N = (N_t)_{(t\geq 0)}$ a Poisson process with intensity λ .

$U = (U_k)_{(k\geq 1)}$ is a sequence of i.i.d. random variables, normally distributed with 0 mean and volatility σ_U .

This model is very tractable and calculations are easy.

References : Merton [1976], Ramezani & Zeng [1998]

Statistical inference

Two-step parameters estimation:

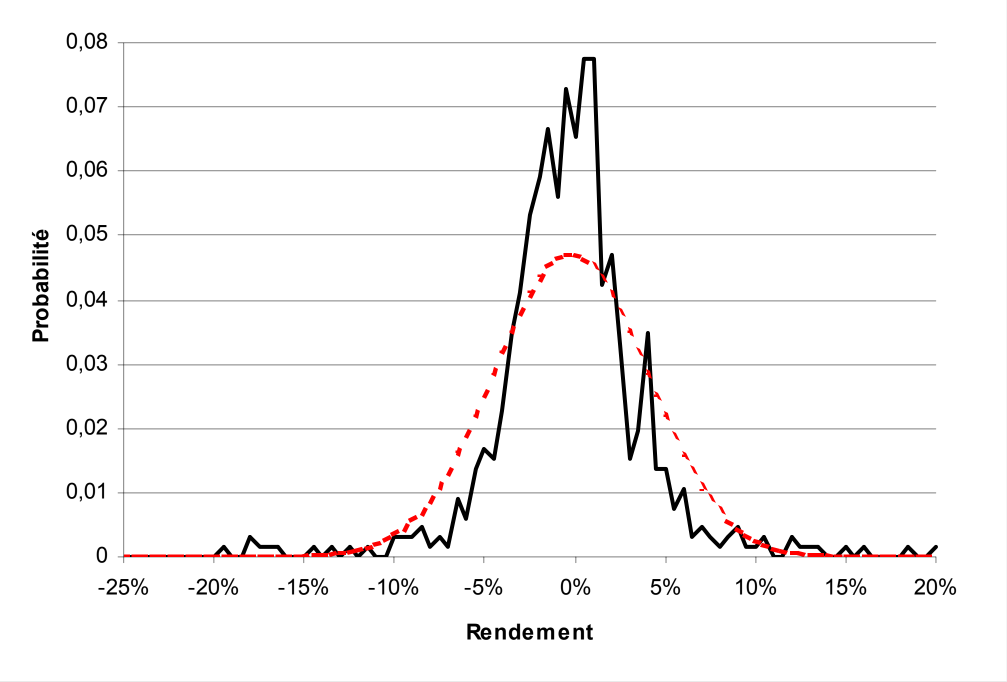
- moment method : $m_{2k} = E(r-m)^{2k} = e^{-\lambda}\frac{(2k)!}{2^k k!}\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\left(\sigma^2 + n\sigma_u^2\right)^k$

- maximum likelihood (the moment estimators are used to initialize the algorithm)

$$L\left(x_1,\ldots,x_n,\mu,\sigma^2,\lambda,\sigma_u^2\right) = \prod_{i=1}^{n}\frac{e^{-\lambda}}{\sqrt{2\pi}}\left[\sum_{n=0}^{\infty}\left[\frac{\lambda^n}{n!\times\sqrt{\left(\sigma^2+n\sigma_u^2\right)}}\times\mathbf{exp}\left(-\frac{\left(x_i-\mu+\frac{\sigma^2}{2}\right)^2}{2\left(\sigma^2+n\sigma_u^2\right)}\right)\right]\right]$$

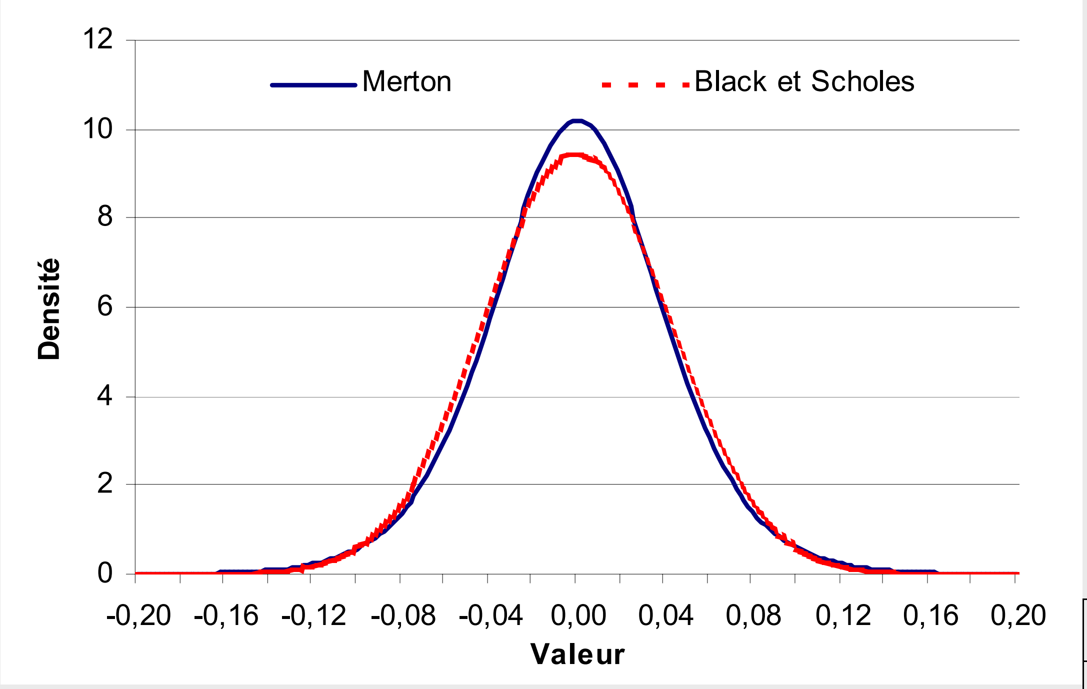
Numerical illustration : Alcatel (1/2)

The $x_i = x(t_i) = \ln \frac{S_{t_i}}{S_{t_{i-1}}}$ are i.i.d. r.v.



The expected value of the underlying stock is 9,07 €, with standard deviation 2,97 ; the asymmetry equals 0,53>0. The return is NOT gaussian.

Numerical illustration : Alcatel (2/2)



50% of the volatility comes from the jump component!

(expected return 5%, volatility 45%)

	Moment estimators	Likelihood estimators
μ	0,00067364	0,00060628
σ^2	0,03095012	0,02785511
λ	0,99328746	0,89395872
σ_u^2	0,02884728	0,03173200

Fair value of a call option (1/2)

In the case of the Mertom's model, martingale measure is not unique and the market is an incomplete market. Various approaches can be adopted to justify the choice of risk measure used to price the option (see BALLOTTA [2004]).

We adopt here the initial solution of Merton consisting in considering that the risk associated with the jump component is non-systematic (i.e. specific to the asset) and may thus be diversified: one does not associate risk premium to it.

That thus resultes in simply evaluating the expected value of associated flows.

Fair value of a call option (2/2)

If the exercise probability is very close in both models, the option fair value is very different.

For example, with S=100, K=110, T=1, a global volatility equals to 25% ; the volatility of the Brownian motion part is 15% and 20% for the jump part.

In this case, the call option price is 9,15 in the B&S model, 5,20 in the Merton model. The exercise probability is 42% in both cases.

→ In an IFRS 4 perspective, this could have a big impact in pricing and reserving a guaranteed minimum death benefits option in a unit-linked insurance contract for example.

Solvency II : target capital (1/4)

The model comes from Deelstra & Janssen [1998] :

The liability : $$L_t = L_0 \exp\left\{\left(\mu_L - \frac{\sigma_L^2}{2}\right)t + \sigma_L B_t^L\right\}$$

The asset : $$A_t = A_0 \exp\left\{\left(\mu_A - \frac{\sigma_A^2}{2}\right)t + \sigma_A B_t^A + \sum_{k=1}^{N_t} U_k\right\}$$

And the "mismatch process" : $$a_t = \ln \frac{A_t}{L_t}$$

$$a_t = a_0 + \left(\mu_A - \mu_L - \frac{\sigma_A^2 - \sigma_L^2}{2}\right)t + \sqrt{\sigma_A^2 + \sigma_L^2}\, B_t + \sum_{k=1}^{N_t} U_k$$

Solvency II : target capital (2/4)

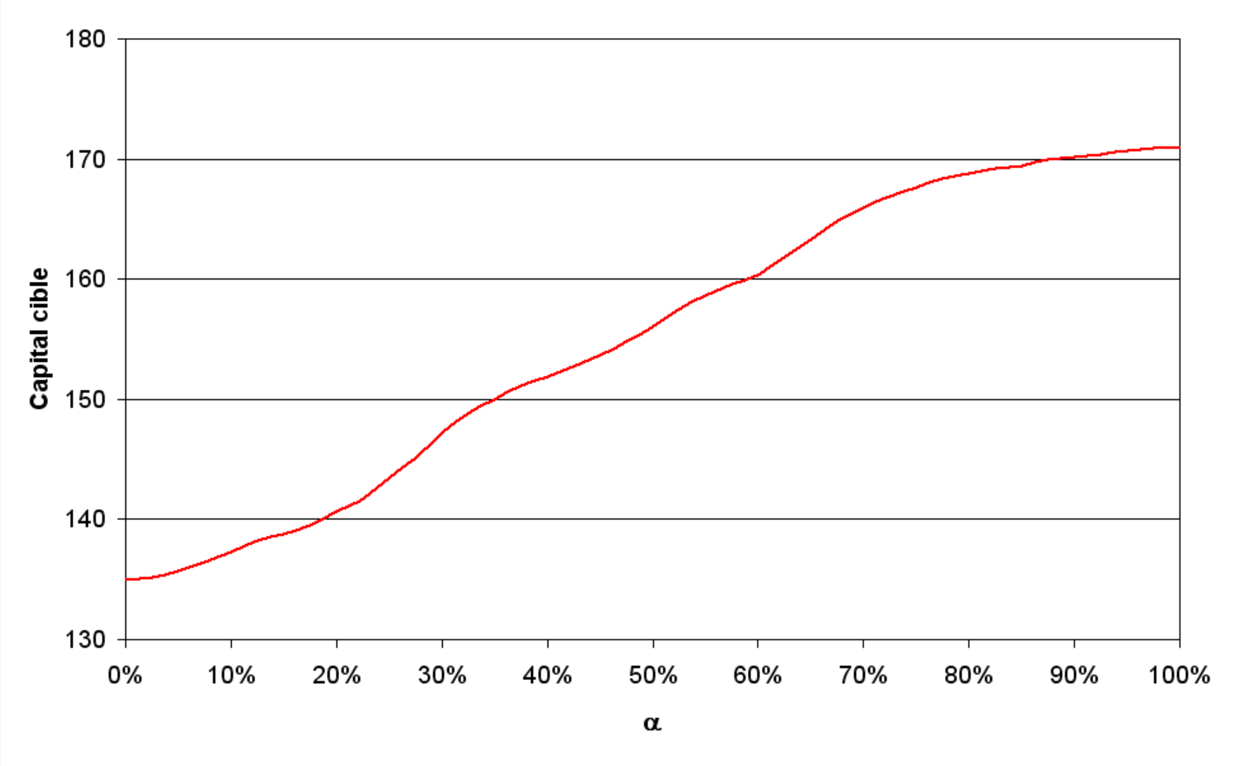
In a context of the type Solvency 2, an insurance company must have a level of own capital (the target capital) which controls the total risk of the company at a predetermined horizon. Let us consider here that the measurement of the total risk of the company is the probability of ruin which is a question of controlling at horizon 1 year with a probability of 1 %.

We suppose that at the end of the year, the company was to have enough capital corresponding to a liability which has to be 100. The insurer has at initial time this amount in provisions, it thus acts to determine the amount of the target capital γ such as:

$$\mathbf{Pr}\left[(100+\gamma)\,\mathbf{exp}\left\{\mu-\frac{\sigma^2}{2}+\sigma B_1+\sum_{k=1}^{N_1}U_k\right\}\leq 100\right]\leq 0{,}01$$

Solvency II : target capital (3/4)

We suppose moreover : $\mu - \frac{\sigma^2}{2} = \ln 0{,}08$ $\lambda = 1{,}5$ $\sigma^2 + \lambda\sigma_U^2 = 0{,}16$



$$\alpha = \frac{\sigma_U \sqrt{\lambda}}{\sqrt{\sigma^2 + \lambda\,\sigma_U^2}}$$

Solvency II : target capital (4/4)

It comes out from this illustration, that the use of the Black & Scholes model leads to underestimate the level of the target capital in a proportion which can be important.

As an example if half of total volatility is explained by the jumps, the target capital resulting from the B&S model underestimates of 13,5 % the true required capital.

Conclusion (1/2)

The Black & Scholes model is a standard used in many practical situations in insurance: option pricing, ALM, determination of the probability of ruin, etc.

The principal quality of this model lies in its simplicity of implementation (calculation of the associated functionals, estimate of the parameters, etc), its adequacy with the data being in general of quite poor quality.

Conclusion (2/2)

The model that we present here, initially suggested by Merton preserves the ease of use, while improving the sensitivity and the adequacy to the data. It is possible to integrate into the asset modelling properties such as asymmetry and a tail of distribution thicker than that of a normal law.

These properties are not without consequence on the appreciation of the level of the capital of solvency within the meaning of Solvency II.

Thus, the reflexions on the standard model of Solvency II (see for example Djehiche and Hörfelt [2004] or Planchet and Thérond [2005]) must integrate this type of model to guarantee a sufficient appreciation of solvency in this new reference framework.

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